CREATIVE MATHEMATICAL REASONING IN MATHEMATICAL LITERACY REVIEW BASED ON PISA 2015 DOMAIN PROBLEM

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Abstract. Mathematics learning in line with 21st-century learning objectives has 4C characteristics: (Communication, Collaboration, Critical Thinking and Problem Solving, Creativity and Innovation). Mathematical material and mathematical reasoning are two inseparable matters, mathematical material understood through reasoning and reasoning understood and trained through learning the mathematical material. There are two types of reasoning that students often use in completing mathematical tasks, namely: Creative Reasoning and Imitative Reasoning. Creative reasoning has four criteria: novelty, flexibility, plausibility and mathematical foundation. Mathematical Literacy is the individual's ability to formulate, employ, and interpret mathematics in various contexts. It includes mathematical reasoning and uses mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. Thus mathematical literacy helps one to recognize the role of mathematics in the world and to make judgments and decisions needed as citizens. PISA measures what the student knows and what he can do (application) with his knowledge of mathematical creative reasoning will examine more deeply the characteristics of mathematical literacy skills in which it contains some characteristics of creative reasoning ability.

Keywords: *Creative Mathematical Reasoning, Mathematical Literacy, Mathematics Framework for PISA*

I. INTRODUCTION

The purpose of mathematics learning are: (1) to train how to understand something through thinking and reasoning to draw a conclusion, (2) to develop the creativity of learners by involving intuition, imagination, and an invention developed from the idea of curiosity, diverging, original, can make predictions or guesses to solve a problem encountered, (3) develop problem-solving skills, and (4) develop the ability to convey information and communicate ideas. Depdiknas (2006) explained that the objectives of mathematics learning are: (a) understanding the concept of mathematics, explaining the interrelationship between concepts and applying concepts or algorithms flexibly, accurately, efficiently and appropriately in problem solving; (b) using reasoning on patterns and traits , performing mathematical manipulations in generalizing, compiling evidence, or explaining mathematical ideas and statements, (c) solving problems that include the ability to understand problems, designing mathematical models, solving models, and interpreting solutions obtained, d) communicating ideas with symbols, tables , diagrams or other media to clarify circumstances or problems, and (e) have an appreciative attitude to the usefulness of mathematics in life that is curiosity, attention, and interest in learning mathematics, as well as a tenacious attitude and confidence in problem solving. So should the development of learning mathematics in the school should consider that mathematics is one means of formation of student mindsets that can be measured by the ability. So it is clear that for students (learners) have good mathematical skills, then the teacher of mathematics as a supporting factor must also have good mathematical skills as well.

One of the abilities that can form a person's mindset is the ability of mathematical reasoning. It is in line with what Killpatrick, Swafford & Findell (2001) finds out that one's ability to solve a problem is the components of the mathematical skill: (1) conceptual understanding; (2) procedural fluency; (3) strategic competence; (4) adaptive reasoning; and (5) productive disposition. Bergqvist (2007) suggests the mathematical reasoning framework described in Figure 1 as follows:

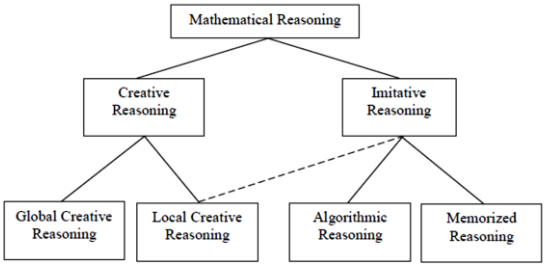

Figure 1. Mathematical Reasoning Framework

In addition, Lithner (2008) suggests that Creative Reasoning (CR) is a reasoning with emphasis on problem-solving process that includes novelty, flexibility, plausible and mathematical foundation.

1. Novelty In creative reasoning, a series of new solutions (for reasoning) are created in problem-solving or a series of forgotten, re-created solutions. The only exemplary answer to the settlement procedure is not included in the creative reasoning.
2. Flexibility (flexibility). Use different approaches and be adapted to the appropriate problem situations.
3. Plausible There are arguments in favor of choice and application of strategy to reinforce the reason that the conclusions are true or reasonable. In this case, guessing the answer is not allowed.
4. Mathematically based (mathematical foundation). The argument given by the reasoner exists in the mathematical intrinsic properties of the components contained in the reasoning.

In everyday life, students deal with issues related to personal, community, work, and scientific. Many of these problems are related to the application of mathematics. Good math mastery can help students solve the problem. The question is what math skills are needed to solve problems in everyday life. Or specifically, what math competencies for 15-year-olds (obtained through special schools or training) are useful for their future careers or for continuing education to the college level. Therefore, the required mathematical literacy is the target of PISA. For PISA 2015, Mathematical Literacy is the individual's ability to formulate, employ, and interpret mathematics in a variety of contexts. It includes mathematical reasoning and uses mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. Thus mathematical literacy helps one to recognize the role of mathematics in the world and to make judgments and decisions needed as citizens (OECD, 2015). Based on several definitions of creative reasoning and mathematical Literacy and its characteristics, it can be seen a deep slice between the creative reasoning ability of mathematics in the ability of mathematical literacy. Therefore, since the problem of mathematical literacy is closely related to the theoretical framework and PISA framework, this article will specifically analyze the domain of the new PISA problem, which is 2015 based on an indicator of mathematical creative reasoning ability

II. METHODOLOGY

The methodology used in this research is a research with case study methods or approach. Case study including in research of descriptive analysis, research years are focused on a certain case to be saved and analyzed similarly until. this research makes self-intensive on an object that learns as a case.

III. THEORETICAL FRAMEWORK

National Adult Literacy Survey (NCES 1993), Science and skills are essential in performing an arithmetic operation, either singly or sequentially, in numbers used in print media such as checkbooks, or used in a paper order. International Life Skill Survey (ILSS 2000): Includes some of the abilities, beliefs, dispositions, habits of mind, communication skills, and problem-solving skills necessary to involve the effectiveness of a situation that arises in life and work. The problem of the above definition is that both tend to place more emphasis on the real context in this case on quantity. Mathematical literacy is not limited to the ability to impose a quantitative aspect of mathematics but also involves mathematics in a broad sense. For example, when there are foreigners who visit a place, the stranger ask is the direction. In general, some people answer the question is not appropriate because most people do not have the ability to show direction/navigation. Another example is in reading the map and its interpretation, spatial consciousness or called "spatial property" (Freudenthal 1973), understanding of planning in building a house, and others. All kinds of visualizations are focused on aspects of mathematical literacy and are an essential component of absolute literacy. This program started in 2000. The results of the existing PISA study are PISA 2000, PISA 2003, PISA 2006, PISA 2009, PISA 2012 and the latest is PISA 2015. Indonesia participated in this PISA program since 2000.

Based on the following table, especially in the field of math and science, Indonesian students from 2009-2015 the value continues to climb, but when compared with other countries Indonesian students are still lacking and still have hard work. Meanwhile, Germans in mathematics and science have the opposite value of decline, but when compared with the results of Indonesia is still much better. In the field of reading ability, Indonesia declined and Germany improved. The best results are achieved by students from Singapore, this time in the results of the 2015 PISA test results show their greatest value for math, reading, and science, perhaps because Shanghai did not participate in this 2015 PISA study. But indeed Singapore in 2009 and 2012 is always behind Shanghai

Table 1. PISA test results

Hasil Uji Studi PISA

Mathematika	PISA 2009		PISA 2012		PISA 2015	
Rangking	Negara	Nilai	Rangking	Negara	Nilai	Rangking	Negara	Nilai
1	Shanghai	600	1	Shanghai	613	1	Singapur	564
2	Singapur	562	2	Singapur	573	2	Hongkong	548
3	Hongkong	555	3	Hongkong	561	3	Macao	544
4	Korea	546	4	Taiwan	560	4	Taiwan	542
5	Taiwan	543	5	Korea	554	5	Japan	532
	Jerman	513		Jerman	514		Jerman	506
	USA	487		USA	481		USA	470
68	Indonesia	371	64	Indonesia	375	63	Indonesia	386
Kemampuan membaca	**PISA 2009**		**PISA 2012**		**PISA 2015**	
Rangking	Negara	Nilai	Rangking	Negara	Nilai	Rangking	Negara	Nilai
1	Shanghai	556	1	Shanghai	570	1	Singapur	535
2	Korea	539	2	Hongkong	545	2	Hongkong	527
3	Finlandia	536	3	Singapur	542	3	Kanada	527
4	Hongkong	533	4	Japan	538	4	Finland	526
5	Singapur	526	5	Korea	536	5	Estonia	519
	Jerman	497		Jerman	508		Jerman	509
	USA	500		USA	498		USA	497
62	Indonesia	402	63	Indonesia	396	64	Indonesia	397
Sains	**PISA 2009**		**PISA 2012**		**PISA 2015**	
Rangking	Negara	Nilai	Rangking	Negara	Nilai	Rangking	Negara	Nilai
1	Shanghai	575	1	Shanghai	580	1	Singapur	556
2	Finlandia	554	2	Hongkong	555	2	Japan	538
3	Hongkong	549	3	Singapur	551	3	Estonia	534
4	Singapur	542	4	Japan	547	4	Taiwan	532
5	Japan	539	5	Korea	538	5	Finland	531
	Jerman	520		Jerman	524		Jerman	509
	USA	502		USA	497		USA	496
66	Indonesia	383	64	Indonesia	382	64	Indonesia	403
	74 negara peserta			65 negara peserta			72 negara peserta	

IV. RESULT AND DISCUSSION

A. The purpose of PISA

PISA orientation reflects changes in curriculum goals and objectives, which pay more attention to what students can do than what they learn in school. Therefore, students are expected to have the ability to literacy (literacy). PISA is designed to collect information through a rotational 3 yearly assessment to find out the literacy of students in reading, math, and science. PISA also provides information on factors that influence the development of students' skills and attitudes both at home and at school as well as assess how these factors

integrate so as to influence the development of a country's policy (OECD, 2010).

Lange, J.D. 2003. Mathematical Literacy Competence (LM):

1. Mathematical Thinking and Reasoning
   - Asking a mathematical question
   - Master the answers to the math questions
   - Differentiate various statements
   - Understand and master mathematical concepts
2. Mathematical Argumentation
   - Knowing the proof
   - Know how the proof differs from other forms of mathematical reasoning
   - Following and appraising from the argument/argument thread
   - Have a sense of an invention
   - Create and show mathematical arguments
3. Mathematical Communication
   - The ability of individuals to communicate orally, writing and other visual forms
   - Understanding others in work
4. Modeling
   - Translating real conditions into mathematical structures
   - Interpret mathematical models in contextual form
   - Working in a model
   - Validate the model
   - Reflection
   - Analysis
   - Conduct a critique of a model or solution
   - Reflections on the modeling process
5. Problem Posing and solving
   - Posing
   - Formulating
   - Defining
   - Problem solving
6. Representation
   - Decoding,
   - encoding,
   - translating,
   - interpreting
7. Symbols
   Using symbols in mathematical operations
8. Tools and Technology
   Using help and tools, including technology as needed.

To facilitate the observers of mathematics, it is necessary to provide one picture of how the framework of Pisa 2015 and how far the educators in Indonesia to evaluate the questions given to learners in facing the challenges of the assessment of Pisa 2018.

In OECD 2015, the mathematical modeling cycle in Figure 1 illustrates the ideal and simplified version of the stage through which the problem solver moves when it shows mathematical literacy. It shows an ideal set of stages that begin with "problems in context." A solver tries to identify the relevant mathematics in problem situations and formulate mathematical situations according to the concepts and relationships identified and simplify the assumptions made. The troubleshooter changed the "problem in a context" to "math problems". The downward arrow in Figure 1 illustrates the work done as a problem solver employing mathematical concepts, procedures, facts, and tools to obtain "mathematical results." This stage usually involves mathematical reasoning, manipulation, transformation, and calculation. Furthermore, "mathematical results" should be interpreted ("results in context"). It involves troubleshooters interpreting, applying, and evaluating mathematical and fair results in the context of real-world problems. This process of formulating, employing, and interpreting mathematics is a key component of the mathematical modeling cycle and also a key

component of the definition of mathematical literacy.

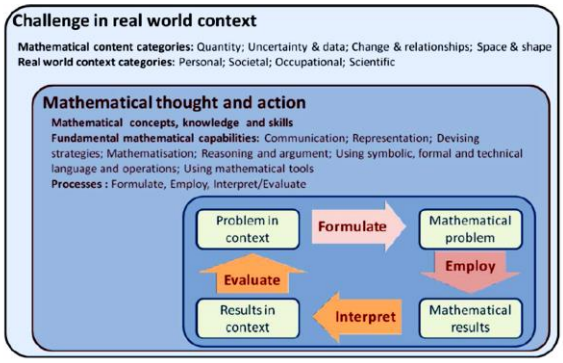

Figure 2. A Model of mathematical literacy in practice

The seven basic math skills used in the Pisa 2015 framework are as follows:

1. Communication:
   Mathematical literacy involves communication. individuals feel some challenges and are stimulated to recognize and understand the problem situation. Reading, decoding and interpreting statements, questions, tasks or objects allows the individual to form a mental model of the situation, which is an important step in understanding, explaining and formulating the problem. During the solution process, the results may need to be summarized and presented. Then, once the solution is found, the problem solver may need to present a solution, and possibly an explanation or justification, to others.
2. Mathematising:
   Literacy Mathematics can involve transforming a defined problem in the real world into a tight form of mathematics (which can include structuring, conceptualizing, making assumptions, and/ or formulating models), or interpreting or evaluating mathematical results or mathematical models in relation to the original problem
3. Representation
   Literacy, mathematics very often involves representations of mathematical objects and situations. It can require selecting, interpreting, translating between, and use various representations to capture situations, interact with problems, or to present a person's work. Such representations include graphs, tables, diagrams, drawings, equations, formulas, and concrete forms.
4. Reasoning and argument:
   The mathematical abilities mentioned throughout the phases and activities associated with different mathematical literacy are referred to as reasoning and argumentation. This capability involves a logical thinking process that explores and elements of relationship problems so as to draw conclusions from them, check the justifications given, or provide justification of the statement or solution to the problem.
5. Develop strategies to solve problems:
   Mathematical literacy often requires formulating strategies for solving mathematical problems. It involves a series of critical control processes that guide an individual to effectively recognize, formulate and solve problems. these skills are characterized as choosing or devising a plan or strategy for using math to solve problems arising from a task or context, and guiding its implementation.
6. Using symbolic, formal and technical languages and operations
   Mathematical Literacy uses symbolic, language and formal and technical operations. this involves understanding, interpreting, manipulating, and making use of symbolic expressions in the context of mathematics (including arithmetic expressions and operations) governed by mathematical conventions and rules. It also involves understanding and utilizing formal constructs based on formal definitions, rules and systems and also using algorithms with

those entities. The symbols, rules and systems used will vary according to what the content of certain mathematical knowledge is required for a particular task to formulate, solve or interpret mathematics.

7. Using mathematical tool

   Final mathematical skills that support mathematical Literacy in practice using mathematical tools. mathematical tools include physical tools such as measuring tools, as well as the more widely available computer-based calculators and tools. This capability involves knowledge in utilizing various tools that can aid in mathematical activity, and know the limits of the tool. mathematical tools can also have an important role in communicating results.

V. RESULT AND DISCUSSION

The following is presented an example of the type of student's answers using mathematical literacy skills, then will be given an analysis of the parts that intersect with the ability of mathematical creative reasoning, referring to Lithner (2006: 10).

A pizzeria serves two round pizzas of the same thickness in different sizes. The smaller one has a diameter of 30 cm and costs 30 zeds. The larger one has a diameter of 40 cm and costs 40 zeds.

Which pizza is better value for money? Show your reasoning.

Figure 3. An Illustrative item Pizzas

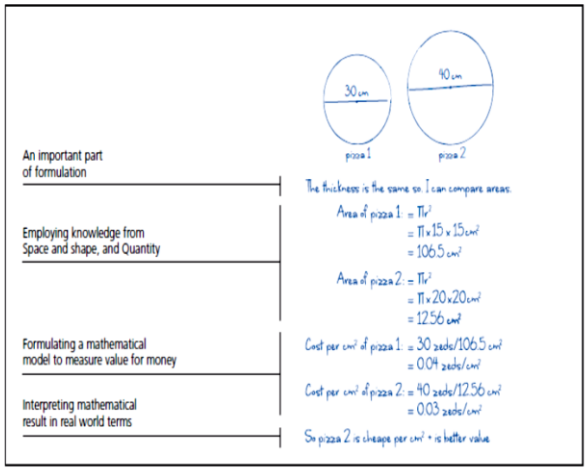

Figure 4. Sample response to Pizzas

Based on the given case example, the analysis of the parts that intersect with the mathematical creative reasoning abilities is as follows

1. Novelty; students use the broad concept of the circle in solving the problem, and already seen the algorithm to get the cheapest price of the two pizzas, in this process the students do not use algorithm procedures given by others
2. Flexible; Students can take the initiative to analyze the situation and adapt it to its condition, this is an unusual initiative among the students.
3. It makes sense; Students have a reasonable mathematical argument about the choice of strategy and its conclusions.
4. Mathematically based; can develop a good conceptual understanding of the area of the circle, the comparison and its application

VI. CONCLUSIONS

Reasoning that students often use in completing mathematical tasks is given broadly divided into two types, namely creative reasoning, and imitative reasoning. Creative reasoning has four criteria: novelty, flexibility, plausibility and mathematical foundation.

The success of Indonesian students in solving the PISA problems is largely determined by the evaluation system and the ability of teachers to develop students' mathematical literacy and mathematical reasoning abilities. Therefore, it is necessary to develop PISA equivalent problems with Indonesian context either by teachers, researchers, or academics in mathematics education.

Based on the results of the above discussion, it is necessary to do a real step in order to reevaluate the performance of observers of mathematics education in the face of the challenges of Pisa 2018.

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